2023

Time :As in Programme

Full Marks : 80

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer the following questions. 1x12

a. The value of $\lim_{x\to 1}\frac{\log(1-x)}{\cot \pi x} =$ ______.

b. The series expansion of cos x is valid for all $x \in R$, True/ False.

c. There exist a function which is continuous at every point but its derivative does not exist any where (true/false)

d. Is the function $f(x) = x^{1/3}$ differentiable at x=0 (yes/No)

e. All Riemann integrable functions on [a,b] are bounded on [a,b]. (Ture/False).

f. Let f(x) = k on [1, 3] then the value of $\int_1^3 f dx =$ 2k.

g. Is the Integral $\int_0^1 \frac{\sin x}{x}$ Improper ? (Yes / No)

h. Is the Integral $\int_0^1 \frac{dx}{\sqrt{1-x}}$ converges ? (Yes/No)

i. A convergent integral which is not absolutely convergent is called ______. conditionally

(Turn Over)

j. Write the condition for 'n; for which the integral $\int_a^\infty \frac{dx}{x^n}$ converges

k. Let $\{f_n\}$ be a sequence of functions where $f_n(x)=\frac{\sin nx}{\sqrt{n}}$,

then $\lim_{n\to\infty}\frac{\sin nx}{\sqrt{n}} =$ ______.

l. Can you say f(x) is continuous if (f_n) is continuous for each 'n' under pointwise convergence, where $f_n \to f$. (Yes /No.)

PART-II

2. Answer any eight within two to three sentences. 2x8

a. Give an example of a function which is convex but not differentiable.

b. Evaluate $\lim_{x\to\infty} e^{-x}.x^2$.

c. Write Taylor's theorem with Lagrange's form of remainder.

d. Define Riemann Sum on [a,b]

e. Define Upper Darboux integral.

f. Give an example of a function which is bounded, but not Riemann integrable.

g. Examine the convergence of $\int_0^\infty e^{-x^2} dx$.

h. Write the comparison Test of limit form for the convergence of $\int_a^b f dx$.

i. Define radius of convergence and interval of convergence of $\sum a n^{x^n}$.

j. Define pointwise Convergence of the sequence of functions.

PART-III

3. Answer any eight of the following (in maximum 75 words.) 3x8

a. Show that $1-\frac{1}{2}x^2 \le \cos x, \forall x \in R$.

b. Define Convex function in ICR. Is f(x) = |x| convex.

c. Write Cauchy's mean value theorem.

d. Show that f(x) = 3x+2 is Riemann integrable on [1,2] and $\int_1^2 (3x+1) = \frac{11}{2}$

e. Prove that of if : [a, b]$\to$R is continuous on [a,b] then f$\in$R[a,b]

f. Let f$\in$B[a,b] then prove that

$$m(b-a) \le \underline{\int_a^b} f dx \le \overline{\int_a^b} f dx \le M(b-a)$$

g. Discuss the convergence of $\int_a^b \frac{\sin x}{x^p} dx$.

h. Find the values of m and n for which the integral $\int_0^1 e^{-mx} x^n dx$ convergs.

i. Let for $0 \le x \le 1$, $f_n(x) = n^2 x(1-x^2)^n$, $n \in N$ Show that $\lim_{n\to\infty} \int_0^1 f_n(x) dx \ne \int_0^1 \lim_{n\to\infty} f_n(x) dx$.

j. State and prove Cauchy Hadamard theorem.

PART-IV

Answer within 500 words each. 7x4

4. Let I be an open interval and let f:I$\to$R have a second derivative on I. Then prove that 'f' is convex function on I if and only if $f''(x) \ge 0, \forall x \in I$.

OR

Evaluate $\lim\limits_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$

5. If f_1 and f_2 are two bounded and integrable functions on [a,b] then prove that $f = f_1+f_2$ also integrable on [a,b] and

$$\int_a^b f dx = \int_a^b f_1 dx + \int_a^b f_2 dx$$

OR

Let $f \in B[a,b]$ then prove that $f \in R[a,b]$ iff for each $\in > 0$ there exist a partition p of [a,b]. Show that $U(f,p) - L(f,b) < \in$.

6. Show that the integral $\int_0^\infty x^{m-1} e^{-x} dx$ is convergent iff $m>0$.

OR

Show that the integral $\int_0^{\pi/2} \log \sin x \, dx$ is convergent and hence evaluate it.

7. Let (f_n) be a sequence of continuous functions on $E \subset C$ converging uniformly to f on E. Then prove that f is continuous on E.

OR

Let (f_n) be a sequence of functions in R[a,b] converging uniformly to f. Then prove that $f \in R[a,b]$ and

$$\lim_{n\to\infty} \int_a^b f_n(x)dx = \int_a^b f(x)dx.$$

$\lim\limits_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$

+3-III-S-CBCS(MS)-Arts/Sc(H)-Core-VI-Math-R&B

2023

Time :As in Programme

Full Marks : 80

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer the following questions. 1x12

   a. Write the number of elements in a dehedral group D_4.

   b. Any two groups of three elements are Isomorphic (True/ False)

   c. If p is a prime number $p|0(G)$ then for $a \in G$, $a^p \notin G$ (True/ False)

   d. Let $H = \{0, \pm 3, \pm 6, \pm 9, ...\}$

      Find all the left cosets of H in Z.

   e. Number of homomorphism from Z_{12} to Z_{30} are _____.

   f. Can an abelian group have a non-abelian subgroup.

   g. A group of orer 4 is abelian write true or false.

   h. A product of disjoint cycle is even if and only if _____.

   i. What is the maximum order of any element in A_{10}.

   j. Find the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$.

k. Every Isomorphic images of a cyclic group is cyclic. (True/False)

l. How many elements of order 5 are there in S_7.

PART-II

2. Answer any eight within two to three sentences. 2x8

a. Define Normalizer of the Group G.

b. Find all the generators of Z_6.

c. Define Dihedral group.

d. Find $Z_2 \oplus Z_3$.

e. Write first isomorphism theorems.

f. Define factor group.

g. If G is a finite group, then prove that $a^{|G|} = e$, for all $a \in G$

h. Express the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 1 & 4 & 8 & 6 & 9 & 7 & 5 \end{pmatrix}$$ as the product of disjoint cycles.

i. List the elements of the subgroup <7> is U(20)

j. If G is a group then prove that for $a \in G$, $(a^{-1})^{-1}=a$.

PART-III

3. Answer any eight of the following (in maximum 75 words.) 3x8

a. Let G be a group and $a \in G$, prove that $\langle a^{-1} \rangle = \langle a \rangle$.

b. Give an example to prove that the converse of the lagrange's theorem is not true.

c. Prove that Intersection of two normal Subgroups of G is again a normal subgroup G.

d. Prove that centre of a group 'G' is a normal subgroup of G

e. Prove that A_n is normal in S_n.

f. Let H be a subgroup of G and let $a \in G$ then prove that $aH = H$ iff $a \in H$.

g. Let H and K are subgroups of a group G and that |H| and |K| are relative prime. Show that $H \cap K = \{e\}$

h. Prove that U_{55} is Isomorphic U_{75}.

i. Prove that a group of prime order is cyclic.

j. If $G \rightarrow \bar{G}$ is a homomorphism and H is a normal subgroup of G, then prove that $\phi(H)$ is normal in $\phi(G)$.

PART-IV

Answer within 500 words each. 7x4

4. Prove that a group G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for $a, b \in G$

OR

Let H be a non-empty finite subset of a group G then prove that H is a subgroup of G iff H is closed under the operation of G.

5. Let G be an abelian group and H and K are subgroups of G Then prove that HK is a subgroup of G

OR

Let G and H be finite cyclic group. Then prove that $G \oplus H$ is cyclic iff |G| and |H| are relatively prime.

6. Define external direct product of a finite number of groups. Prove that the external direct product of finite number of groups is a group.

OR

State and prove Lagrange's theorem for groups.

7. State and prove Cauchy's theorem for abelian groups.

OR

If ϕ is a homomorphism of G into $\overline{G}$ then show that

a. $\phi(e)=\overline{e}, e\in G$

b. $\phi(x^{-1})=[\phi(x)]^{-1}$

4 7×6×5×4×3

42
5
210
4
840
3
20

2023

Time :As in Programme

Full Marks : 60

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer the following questions. 1x8

a. $e^y u_y - xyv_x = xz^2$ is a semi-linear PDE. (True/Fasle)

b. The PDE formed from the equation $z=e^{mx}f(x+y)$ is _____.

c. The solution of $xp + yq = z$ is _____.

d. A solution of the PDE $\frac{\partial^2 u}{\partial^2 x^2} - 9\frac{\partial^2 u}{\partial y^2} = 0$ is _____.

e. Find the region for which the PDE $y^3 U_{xx} - (x^2-1)U_{xy} = 0$ is hyperbolic.

f. The D' Alembert's solution of the one-dimensional wave equation if the string is at rest is _____.

g. The number of independent constant in the general solution of the system $\frac{dx}{dt} + \frac{dy}{dt} = e^t, \frac{dx}{dt} + \frac{dy}{dt} - t = 0$ is _____.

h. A non-trival solution of the system $\frac{dx}{dt} = 3x - y, \frac{dy}{dt} = 4x - y$ is _____.

(Turn Over)

PART-II

2. Answer any eight within two to three sentences. 1.5x8

a. Form the PDE from $z = ax+by$.

b. Write the canonical form of a second order parabolic PDE.

c. State different domain for which the equation $v_{xx} + xv_{yy}=0$ can be classified as elliptic.

d. What is the most general form of a quasilinear PDE.

e. Find the characteristics and characteristic co-ordinates of the equation $2U_{xx} - 4U_{xy} + 2U_{yy} + 3U=0$.

f. Is the equation $x^2U_{xx}+2xyU_{xy}+y^2U_{yy}=0$ is parabolic everywhere

g. If $U = x^2+y^2$, then Test whether it is a solution of a laplace equation.

h. Use operator method to find the solution of the linear system

$$x' + y' - x - 3y = e^t$$

$$x' + y' + x = e^{3t}$$

i. Write down the complete integral of $q = 3p^2$.

j. Write a mathematical model representating Semi-infinite string with free end.

PART-III

3. Answer any eight of the following (in maximum 75 words.) 2x8

a. Find the PDE representing all spheres whose centre lie on z - axis.

b. Solve $\frac{\partial u}{\partial x} = 6\frac{\partial u}{\partial t} + u$ using method of separation of variable if $u(x,0) = 10e^{-x}$.

c. Write the necessary conditions to solve the heat equation.

d. Write the D' Alemberts' solution for the string has initial conditions $U(x,0) = e^x$, $U_t(x,0)=0$.

e. A string is streached and fastened to two point 'l' apart. Motion is started by displacing the string into the form $y=y_0 \sin\frac{\pi x}{l}$ from which it is released at $t = 0$. Formulate this problem as boundary value problem.

f. Obtain the general solution of $3U_{xx} + 10u_{xy} + 3U_{yy} = 0$.

g. Obtain the General solution of the Cauchy problem $U_{tt} - 9U_{xx} = 0$. $U(x,0)=\cos x$, $U_t(x,0) = \sin 2x$, $x \in R$

h. Prove that the characterstic equation of one-dimensional wave equation is a straight line.

i. Solve $(D^2-D'^2)z = 0$

j. Verify that $x = 3e^{7t}$, $y = e^{7t}$ and $x=e^{-t}$, $y = -2e^{-t}$ are solutions of $\frac{dx}{dt} = 5x+3y, \frac{dy}{dt} = 4x+y$

PART-IV

Answer within 500 words each. 6x4

4. Obtain the solution of the equation $(y-u)u_x + (u-x)u_y=x-y$

OR

Reduce the equation $v_x-v_y=v$ and $y v_x+v_y = x$ to canonical form and obtain the general solution.

5. Derive one dimensional wave equation.

OR

Show that the equation $4U_{xx} + 5U_{xy} + U_{yy} + U_x + U_y = 2$ is hyberbolic and reduce it to canonical form and solve it.

6. Let the semi - infinite string problem with free end is given by

$V_{tt} = C^2 V_{xx}$, $0<x<\infty$, $t>0$

$U(x,0)=f(x)$, $0 \leq x < \infty$

$U_t(x,0) = g(x)$, $0 \leq x < \infty$

$V_x(0,t) = 0$, $0 \leq t < \infty$

Find the general solution the above problem.

OR

Find the solution of the heat conduction problem

$U_t = KU_{xx}$, $0<x<e$, $t>0$

$U(0,t) = 0$, $t \geq 0$

$U(e,t) = 0$, $t \geq 0$

$U(x,0) = f(x)$, $0 \leq x \leq e$

7. Solve the system

$$2\frac{dx}{dt} - 2\frac{dy}{dt} = 3x + t$$

$$2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y = 2$$ by operator method.

OR

Use method of successive approximation to find the first three terms of a sequence of function that approaches to the exact solution of $\frac{dy}{dx} = xy$; $y(0)=1$.

2023

Time :As in Programme

Full Marks : 60

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer the following questions. 1x8

   a. Newton Rings are due to interference by division of ____.

   b. Bending of light rays at the corner of an obstacle or slit is known as ____.

   c. Which phenomenon predicts the particle nature of radiation ?

   d. Balmer series belongs to ____ region.

   e. The probability density for a stationary state is ____ of time.

   f. The zero point energy of a particle in an one dimensional infinite potential well is ____.

   g. Is velocity of a particle a Galilean invariant ? (yes/no)

   h, Half life = ____ X Mean life.

PART-II

2. Answer any eight within two to three sentences. 1.5x8

   a. Define coherent sources.

   b. State Brewster's law.

c. Show that rest mass of photon is zero.

d. How does Bohr model of an atom explain the stability of the atom ?

e. Write the physical significance of wave function.

f. Why the minimum energy of a particle in a box cannot be zero ?

g. Explain violation of conservation of energy.

h. What is saturation of nuclear force ?

i. Find the decay constant of CS-137. (Halt life = 30 years)

j. Why fusion can only take place at high temperature ?

PART-III

3. Answer any eight of the following (in maximum 75 words.) 2x8

a. Explain Relativistic mass energy relation.

b. Define Mass defect and write its relation with Binding energy.

c. Distinguish between primary and secondary Rainbow.

d. Explain Double Refraction.

e. What is stopping potential ? Does it depend on the intensity of incident radiation ?

f. Find <Px> and <Hx>

g. State basic postulates of special theory of relativity.

h. Explain probability current density.

i. Newton's ring are observed in reflected light of wave length 6000 $\mathring{A}$. Diameter of 10th darking is 0.5cm. Find the radius of curvature.

j. State de-Broglie's hypothesis and explain.

PART-IV

Answer within 500 words each. 6x4

4. With neat ray diagram describe the formation and necessary theory of secondary Rainbow.

OR

Give Huygen's theory of double refraction in an uniaxial crystal. Also discuss about the electromagnetic theory of double refraction.

5. What is compton effect ? Obtain an expression for compton shift due to scattering of photon from electron.

OR

With basic postulates describe the Bohr's theory of Hydrogen atom.

6. State and prove Ehrenfest's Theorem.

OR

Derive and interprete the equation of continuity connecting the probability current density.

7. State and explain the law of radioactivity. Then derive an expression for decay constant.

OR

Derive an expression for Lorentz transformation equations.

+3-III-S-CBCS(MS)-Arts/Sc/Com(H&P)-AECC-E&V-III (R&B)

2023

Time :As in Programme

Full Marks : 25

The figures in the right-hand margin indicate marks.

*Answer **all** the questions*

GROUP-A

1. Answer any five questions. 1x5

ଯେକୌଣସି ପାଞ୍ଚଟି ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ ।

a. Give one example of Drug ?

ପ୍ରାକୃତିକ ନିଶାଦ୍ରବ୍ୟର ଗୋଟିଏ ଉଦାହରଣ ଦିଅ ।

b. What kind of drugs damage the one of human body.

କେଉଁ ପ୍ରକାର ନିଶା ସେବନ ମଣିଷ ଶରୀରରେ ଆସି ନଷ୍ଟର କାରଣ ହୋଇଥାଏ ?

c. Give one example of Addiction ?

ଆସକ୍ତିର ଏକ କାରଣ ଦର୍ଶାଅ ।

d. What type of thing / matter is Drug ?

ନିଶାଦ୍ରବ୍ୟ ଏକ ପ୍ରକାର କେଉଁ ଜାତୀୟ ପଦାର୍ଥ ।

e. What is the chemical when mixed with alcohol can cause of death of a human being ?

କେଉଁ ରାସାୟନିକ ଦ୍ରବ୍ୟ ମିଶ୍ରଣରେ ମଦ୍ୟପାନ ମନୁଷ୍ୟ ମୃତ୍ୟୁର କାରଣ ହୋଇଥାଇପାରେ ।

f. Give one example of approved drug.

ଅନୁମୋଦିତ ନିଶାଦ୍ରବ୍ୟର ଏକ ଉଦାହରଣ ଦିଅ ।

GROUP-B

2. Answer any five questions. 2x5

ନିମ୍ନଲିଖିତ ଯେକୌଣସି ପାଞ୍ଚଟି ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ ।

a. What are the effects of drug addiction ?

ନିଶା ଆସକ୍ତିର କୁପ୍ରଭାବ କ'ଣ ଉଲ୍ଲେଖ କର ।

(Turn Over)

b. Explain the physical symptoms of alcohol addiction.
ମଦ୍ୟପାନ ଆସକ୍ତିର ଶାରୀରିକ ଲକ୍ଷଣ ଉଲ୍ଲେଖ କର ।

c. Give functions of Narcotic control Bureau briefly.
Narcotic control Bureauର ଦୁଇଟି କାର୍ଯ୍ୟାବଳୀ ଉପରେ ସଂକ୍ଷେପରେ ଉଲ୍ଲେଖ କର ।

d. What is the importance of family is social life ?
ସାମାଜିକ ଜୀବନରେ ପରିବାରର ମହତ୍ୱ କ'ଣ ?

e. What are the health problem caused due to drug addiction ?
ନିଶାସକ୍ତତା ଯୋଗୁଁ କେଉଁ ସବୁ ସ୍ୱାସ୍ଥ୍ୟ ସମସ୍ୟା ଦେଖାଯାଏ ?

f. What are the important role of NGOs ?
NGOମାନଙ୍କର ମୁଖ୍ୟ ଭୂମିକା ସବୁ କ'ଣ ?

GROUP-C

Answer any two questions. 5x2
ଯେକୌଣସି ଦୁଇଟି ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ ।

3. Discuss the causes behind the rise of alcohol use in India ? How they can be controled ?
ଭାରତରେ ମଦ ନିଶାର ବ୍ୟବହାର ବୃଦ୍ଧିର କାରଣ ସବୁ ଉଲ୍ଲେଖ କର । ଏହାକୁ କିପରି ରୋକାଯାଇପାରିବ ?

4. Discuss the Myth about alcohol adiction and rehabilitation.
ମଦ୍ୟପାନ ନିଶାସକ୍ତତା ବିଷୟରେ ଲୋକ ଧାରଣା ସବୁ ଉଲ୍ଲେଖ କର । ଏହା କିପରି ଦୂର ହେବ ?

5. Explain the different steps of alcoholism. How to save the youth from addiction ?
ମଦ୍ୟ ନିଶାସକ୍ତତାର ବିଭିନ୍ନ ସ୍ତର / ପର୍ଯ୍ୟାୟ ସବୁ ଉଲ୍ଲେଖ କର । ଯୁବପିଢ଼ିଙ୍କୁ କେମିତି ସୁରକ୍ଷା ଦିଆଯାଇପାରିବ ?

6. Explain the steps that can be taken for a Tobacco as smoke free campuses.
ମହାବିଦ୍ୟାଳୟ ପରିସରକୁ ନିଶାମୁକ୍ତ ପରିବେଶ କରିବା ପାଇଁ କ'ଣ ସବୁ ପଦକ୍ଷେପ ନିଆଯିବା ଉଚିତ ଉଲ୍ଲେଖ କର ।

7. Write down the role of NGOs in a country social development.
ଦେଶର ସାମାଜିକ ବିକାଶରେ NGOମାନଙ୍କର ଭୂମିକା ଉଲ୍ଲେଖ କର ।