+3-III-S-CBCS(MS)-Arts/Sc(H)-Core-V-Math-R&B

2024

Time :As in Programme

Full Marks : 80

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer all questions. 1x12

a. Does L. Hospital rule is applicable to the limit

$$\lim_{x\to\infty}\frac{x^2+2x}{x^3+x}$$ (Yes / No)

b. The function $f(x)=|x| + |x-1|$ is continuous but not differentiable at $x=0$ is it (True/False)

c. $$f(x)=\begin{cases}1, & x\in Q\\ 0, & x\in R-Q\end{cases}$$ is discontinuity of second kind at every point on R is (True/False)

d. $f(x)=x^2$ is uniformly continuous on [a,b] but not on [a, ∞), a>0, is it (true / False)

e. Every monotonic function is Riemann integrable is it (True/False)

f. Define norm of the partition ?

g. Give an example of a function. Which is Reimann integrable but not Monotonic ?

h. The gamma function $\Gamma(p)=\int_0^\infty e^{-t}t^{p-1}dt$ convergence for p = ____.

(Turn Over)

i. Give an example of two functions f and g are not differentiable but f + g is differentiable.

j. $\int_1^\infty \frac{dx}{x^p}$ converges if P = ____.

k. Write statement of Darboux Theorem ?

l. Give an example of a function. Which is continuous but not uniformly continuous ?

PART-II

2. Answer any eight questions. 2x8

a. Prove that $\int_0^\infty e^{-t^2} dt = \frac{1}{2}\sqrt{\pi}$

b. Evaluate $\lim_{x \to 0} \frac{1-\cos x}{x^2}$

c. State Taylor's Theorem ?

d. Show that the series $\sum_{k=2}^{\infty} \frac{1}{k \log k}$ is divergent.

e. If $f(x) = C$ on [a,b] show that $f \in R[a,b]$

f. Test whether $\int_0^1 \frac{\sin x}{\sqrt{x}}$ is absolutly convergent ?

g. Write Maclaurin's Theorem with lagrange's form of remainder.

h. Give an example of a function 'f' which is integrable but $|f|$ is not integrable.

i. Obtain the taylor's expansion of e^x ?

j. Show that $\int_1^\infty \frac{\sin x}{x^p}$ converges absolutly for P>1 ?

PART-III

3. Answer any eight questions. 3x8

a. Evaluate the limit $\lim\limits_{x\to 0}\dfrac{x^2-\sin^2 x}{x^4}$

b. Evaluate $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{\sqrt{n^2-k^2}}{\pi^2}$

c. Show that $\sum\limits_{x\in R} r^n\cos^n x,\ 0<r<1$ convergence uniformly

d. Show that $\int\limits_{2}^{3} f dx = 6$ for $f(x)=2x+1$

e. Find radius of convergence of $\sum\limits_{n=2}^{\infty}\dfrac{z^n}{n(\log n)^2}$?

f. If b>a prove that $\left|\int\limits_{a}^{b}\dfrac{\sin}{x}\right|<\dfrac{2}{a}$?

g. Show that $\int\limits_{0}^{1}\left(2x\sin\dfrac{1}{x}-\cos\dfrac{1}{x}\right)dx=\sin 1$

h. Examine the convergence of integral $\int\limits_{0}^{\pi}\dfrac{dx}{\sin x}$

i. Show that $\int\limits_{-\infty}^{\infty}\dfrac{1}{x^2+1}dx=\pi$

j. Is $\sum\dfrac{1}{n^p+n^q\cdot x^2}$ is uniformly convergence for all real values of x if $P>1$.

PART-IV

Answer all questions. 7x4

4. Expand sin x in power of $(x-\pi/4)$ with help of Tayloris Theorem.

OR

Show that the largest rectangle inscribed in a circle is a square ?

5. Show that every contineous function on $[a,b]$ is integrable.

OR

If f is bounded and integrable on $[a,b]$ then $|f|$ is also bounded and integrable on $[a,b]$ then prove that

$$\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|\,dx$$

6. Show that $\Gamma\left(n+\frac{1}{2}\right)=\frac{(2n)!\sqrt{\pi}}{4^n n!}$ for n = 0, 1, 2, 3 ...

Then find $\Gamma(3/2)$ and $\Gamma(5/2)$

OR

Show that the improper integral.

$I=\int_1^\infty \frac{\sin t}{t^p}dt$ is convergence if P>1.

7. State and prove Cauchy Hadamard Theorem ?

OR

Suppose (f_n) is a sequence of functions defined on E and $|f_n(x)| \leq M_n$ for all $x \in E$ and $n \in N$ where M_n is a sequence of positive, constants the series $\sum_{n=0}^{\infty} f_n(x)$ converges absolutly and uniformly on E if $\sum M_n$ convergence.

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2024

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*Answer **all** questions.*

PART-I

1. Answer all questions. 1x12

 a. Write symmetry of circle ?

 b. Define center of a group ?

 c. Every group has exactly two improper subgroup. (True/False)

 d. Is Every cyclic group is abelian ? (True/False)

 e. If $a \in G$ and $|a|=10$ find $|a^{37}|$

 f. What is the order of the product of a pair of disjoint cycles of length 4 and 6.

 g. Every function is a permentation iff it is one to one is it (True/False)

 h. Z is Isomorphic to Z/nZ the statement is True or False.

 i. A group of prime order is not cyclic, is the statement True (Yes/No)

 j. Find order of (2,3) in $Z_6 \oplus Z_{15}$

k. If $G=\cup(32)$ and $K = \{1,15\}$ Then $|G/K|=8$ is True or False. 3.

l. Define Kernel of homomorphism.

PART-II

2. Answer any eight questions. 2x8

a. Show that $3Z / 12Z \approx Z_4$

b. Let $\phi: G \rightarrow \overline{G}$ be a group homomorphism of G to $\overline{G}$ show that if G is abelian Then $\overline{G}$ is abelian.

c. Prove that any group $|xa\overline{x}| = |a|$.

d. Prove that the dihedral group of order 6 does not have a subgroup of order 4.

e. Show that $Z_2 \oplus Z_2 \oplus Z_2$ has seven subgroup of order 2

f. Let N and 'K' be subgroup of a finite group G and N is normal in G show that $|KN| = \dfrac{|K| \cdot |N|}{|K \cap N|}$

g. ϕ is homomorphism G into $\overline{G}$ prove that $\phi(e) = \overline{e}$

h. If a, b $\in$ G, G be a group and $|a| = 5$ and $a^3 b = ba^3$ show that ab = ba.

i. Express $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$ as product of 2 - cycles.

j. How many elements of order 9 does $Z_3 \oplus Z_9$ have ?

PART-III

3. Answer any eight questions. 3x8

a. Let G be a group and Let $a \in G$ prove that $C(a) = C(a^{-1})$

b. If every element of group G is its own inverse. Then G is abelian prove it.

c. show that $U_{(20)}$ is not cyclic ?

d. Find the generator of an infinite cyclic grlup.

e. Give lattic diagram of sub groups Z_{100}.

f. Show that $U_{(8)} \approx U_{(12)}$

g. Find the right cosets of 4Z in Z.

h. Prove that G is a group of order P^2 and G is not cyclic then $a^p = e$ for all $a \in G$.

i. Every subgroup of an abelian group is normal.

j. Let ϕ be a group homomorphism from G to $\bar{G}$ then Ker ϕ is a normal subgroup of G.

PART-IV

Answer all questions. 7x4

4. Show that a group G is abelian if $(a \cdot b)^i = a^i b^i$ for any three consecutative integers i and a, b $\in$ G

OR

For each 'a' in a group G. Then centralizer of 'a' is a subgroup of G.

(Turn Over)

5. Prove that every cyclic group is cyclic ?

OR

Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

6. Every group is isomorphic to a group of permutation.

OR

State and prove lagrange's theorem ?

7. A subgroup H of a group 'G' is normal subgroup of G if and only if the product of two right cosets of H in G is again a right coset of H in G.

OR

Let G be a finite abelian group and Let 'P' be any prime such that P divides order of G, then G has an element of order P.

+3-III-S-CBCS(MS)-Arts/Sc(H)-Core-VII-Math-R&B

2024

Time :As in Programme

Full Marks : 60

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer all questions. 1x8

a. Write Euler Tricomi's Equation ?

b. Write condition when P.D.E. is parabolic ?

c. Write an example of one dimensional heat Equation ?

d. Write characteristic equation of $yu_y - xu_x = 1$

e. Which three equations are known as Fundamental equation of mathematical physics ?

f. Write canonical form of Euler equation in Elliptic form ?

g. The laplace equation is elliptic type is true or false.

h. Write an example of a 1st order non linear P.D.E.

PART-II

2. Answer any eight questions. 1.5x8

a. Write the general 2nd order Linear P.D.E. in 'n' independent variable.

(Turn Over)

b. Eleminate arbitary function 'f' from

$$Z = y^2 + 2f\left(\frac{1}{x}\log y\right)$$

c. Examine whether x^2uxx - $2xy\ uxy$ + y^2uyy = e^x is parabolic or hyperbolic

d. Define singular solution ?

e. Find general solution of υx=0

f. Find partial differential equation of the surface

$Z=x^n\ f(y/x)$

g. Write an example of intial value problem of semi infinite string with a fixed end ?

h. Find the general solution of $yu_y - xu_x = 1$

i. Write D'Alembert's solution of the Cauchy problem.

j. Define characteristic curve ?

PART-III

3. Answer any eight questions. 2x8

a. Write a mathematical model represting (assumption) vibration of finite string with fixed end

b. Find solution of intial value problem

$U_{tt} - C^2u_{xx} = 0\ x\in\ R,\ t>0,\ u(x,0) = 0$

$U_t\ (x,0)=1$

c. Find the longitudinal ossillation of a rod subjected to the initial condition $u=(x,0) = \sin x\ \ u_t(x,0) = x$

d. Find general solution of $U_{xx} + U_x = 0$

e. Eliminate the arbitary constant a and b from the P.D.E. $Z = (x^2+a)\ (y^2+b)$

f. Find the steady state solution u(x,t) of heat equation $u_t = a^2 u_{xx}$

g. Determine the solution of $U_{xx} - U_{yy} = 1,\ U(x,0) = \sin x,$ $U_y(x,0) = x$

h. Classify the equation as hyperbolic, parabolic or elliptic

$u_{xx} - 4\ u_{xy} + 4u_{yy} = e^y$

i. Write a mathematical model representing semi finite spring with free end.

j. Find characteristic equation of the system $\frac{dx}{dt} = 3x - y$

and $\frac{dy}{dt} = 4x - y$

PART-IV

Answer all questions. 6x4

4. Solve the equation $xZ_x + yZ_y = z$ and find the curve which satisfy the associated characteristic equations and intersect the helix

$$x^2+y^2 = a^2,\ z = b\tan^{-1}\left(\frac{y}{x}\right)$$

OR

Obtain the solution of the equation $x_{ux} + y_{uy} = xe^{-u}$ with data $u = 0$ on $y = x^2$

5. Find general solution of the equation

$U_{xx} + U_{yy} - 2u_{yy} - 3u_x - 6u_y = 9(2x-y)$

OR

Find the general solution of the equation

$4U_{xx} + 5U_{xy} + U_{yy} + U_x + U_y = 2$

6. Determine the solution of the intial boundary value problem

$U_{tt} = 16U_{xx}\ 0<x<\infty,\ t>0$

$U(x,0)=f(x)\ 0\leq x<\infty$

$U_t(x,0) = x\ 0\leq x<\infty$

$U(0,t) = 0,\ t\geq 0$

OR

Obtain the solution of neat flow equation $\frac{\partial u}{\partial t} = c^2 U_{xx}$

7. Find general solution of the system

$$\frac{dx}{dt} = 3x+2y, \frac{dy}{dt} = 5x+3y$$

OR

Find general solution of $\frac{dx}{dt} + 4x + 3y = t$ and

$$\frac{dy}{dt} + 2x + 5y = e^t$$